

# Gribov Problem and BRST Symmetry <sup>†</sup>

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## Abstract

After a brief historical comment on the study of BRS(or BRST) symmetry , we discuss the quantization of gauge theories with Gribov copies. A path integral with BRST symmetry can be formulated by summing the Gribov-type copies in a very specific way if the functional correspondence between  $\tau$  and the gauge parameter  $\omega$  defined by  $\tau(x) = f(A_\mu^\omega(x))$  is “globally single valued”, where  $f(A_\mu^\omega(x)) = 0$  specifies the gauge condition. As an example of the theory which satisfies this criterion, we comment on a soluble gauge model with Gribov-type copies recently analyzed by Friedberg, Lee, Pang and Ren. We also comment on a possible connection of the dynamical instability of BRST symmetry with the Gribov problem on the basis of an index notion.

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# 1 Introduction

The BRS(or BRST) symmetry and its applications to gauge field theories in general have been developed and completed by the efforts of many authors. The Faddeev-Popov ghost fields[1] and the so-called Slavnov-Taylor identities[2] paved a way to the discovery of the BRST symmetry[3]. The Nakanishi-Lautrup  $B$ -field[4], which was introduced to define the Landau gauge in QED, played an important role to define the off-shell closure of the BRST algebra. The BRST symmetry has completely changed the picture of perturbative renormalization of Yang-Mills fields[5]. The path integral formalism[6] as well as anti-field anti-bracket formalism[7] extended the Dirac's general idea of the quantization of singular Lagrangians[8] to the maximum extent. These developemtns have been summarized in several review articles[9] and a text book[10].

As for the study of BRST symmetry in Japan<sup>\*</sup>, the first significant contribution to the subject was made by Kugo and Ojima[11], who utilized the BRST charge to extract the positive metric Hilbert space from an indefinite metric theory containing Faddeev-Popov ghost fields and the time component of gauge field. Their formalism completed a previous attempt toward this direction[12]. The work of Kugo and Ojima was immediately and highly appreciated by two leading Japanese theorists, K. Nishijima and N. Nakanishi, both of whom applied the idea to the formal analyses of Einstein gravity[13]. It was then pointed out[14] that the basic mechanism of selecting the positive metric space in the BRST approach is the BRST cohomology, although at that time this terminology was not known: By using the notion of BRST superfield, a simple and explicit classification of BRST invariant states was given. The basic tone in Japan was an emphasis on the operator treatment of the BRST symmetry rather than on the path integral analysis[15].

In the meantime, the path integral treatment of covariant string theory by Polyakov appeared in 1981[16]. I applied the BRST symmetry to the string theory partly as an application of the path integral treatment of anomalies[17]. The Virasoro condition[18] was then recognized as a result of BRST invariance. It was also noted that the *proper part* of BRST symmetry is preserved for any (target) space-time dimensions, which is related

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<sup>\*</sup>Since many speakers of the Symposium commented on some historical account of their encounter with the BRST symmetry, this short account of mine was added in this written version.

to non-critical string theory. The ghost number anomaly in string theory

$$\partial_\mu j_{gh}^\mu = -\frac{3}{4\pi}\sqrt{g}R \quad (1.1)$$

was first encountered there[19]; this anomaly, which contains the scalar curvature in the right-hand side, was later recognized as a local version of the Riemann-Roch theorem and as such it naturally played an important role in string theory and topological field theories. In the context of the Gribov problem to be discussed below( in particular, the vanishing Faddeev-Popov determinant), the ghost number anomaly in (1.1) generally shows the appearance of the zero modes of ghost fields; for example, 3 ghost zero modes on a sphere. It is known that the appearance of those ghost zero modes does not spoil the BRST symmetric formulation of string theory, at least in the first quantization of strings. This is analogous to the zero modes of fermions in the background of instantons. The mere appearance of zero modes of the Faddeev-Popov determinant thus does not necessarily imply the difficulty of BRST symmetric formulation. More about this will be commented on in Section 4.

About 6 months later since my preprint on the BRST formulation of string theory appeared , Kato and Ogawa , both of whom were graduate students at Kyoto University at that time , performed a very detailed BRST operator analysis of the bosonic string theory[20]. It is well known that their paper greatly influenced the later developments in the covariant treatment of strings and string field theories[21]. The BRS or BRST symmetry thus became a standard terminology of a wider class of particle theorists, as was noted by J. Gomis at this Symposium.

Coming to a fundamental aspect of BRST symmetry, I would like to comment on one of the possible physical interpretations of the famous factor 1/2 in the BRST transformation law of ghost fields<sup>†</sup>, on the basis of the BRST invariance of the path integral measure. This property is seen particularly clearly if one looks at the gravitational BRST symmetry. The structure “constant”  $F_{\alpha\beta}^\mu$  for the coordinate transformation is defined by

$$\begin{aligned} [\xi^\rho(x)\partial_\rho, \eta^\lambda(x)\partial_\lambda]\phi(x) &= (\xi^\alpha\partial_\alpha\eta^\mu - \partial_\beta\xi^\mu\eta^\beta)\partial_\mu\phi(x) \\ &\equiv F_{\alpha\beta}^\mu\xi^\alpha\eta^\beta\partial_\mu\phi(x) \end{aligned} \quad (1.2)$$

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<sup>†</sup>This issue was raised by R. Stora at the Symposium.

The BRST transformation of ghost fields is then defined with a factor of  $1/2$

$$\begin{aligned} c^\mu(x, \theta) &= c^\mu(x) + i\theta \frac{1}{2} F_{\alpha\beta}^\mu c^\alpha(x) c^\beta(x) \\ &= c^\mu(x) + i\theta c^\rho(x) \partial_\rho c^\mu(x) \end{aligned} \quad (1.3)$$

We here used the superfield notation where the second component proportional to the Grassmann number  $\theta$  stands for the BRST transform of the first component. The transformation law in (1.3) does not appear to have an explicit connection with the general coordinate transformation, if one identifies  $i\theta c^\rho(x)$  with the parameter of coordinate transformation, as is the case in metric variables. However, if one considers the differential of the superfield in (1.3), one obtains

$$dc^\mu(x, \theta) = dc^\mu(x) + i\theta [c^\rho(x) \partial_\rho dc^\mu(x) - \partial_\rho c^\mu(x) dc^\rho(x)] \quad (1.4)$$

This expression, when regarded as a transformation law of  $dc^\mu(x)$ , has the structure of general coordinate transformation. Without the factor of  $1/2$ , the differential (1.4) would not have the form of coordinate transformation. It is shown that this last structure, when combined with BRST transformation law of metric variables, is crucial to establish the BRST invariance of the gravitational path integral measure without having an artificial anomaly (or Jacobian)[22]. Note that the Jacobian for the integral measure is defined by (1.4).

Although the success of the BRST symmetry is impressive in the realm of perturbation theory of gauge fields in general, the future prospect of the BRST symmetry in connection with non-perturbative analyses is not quite bright, as was emphasized by C. Becchi at this Symposium[23]. This is due to the presence of the so-called Gribov problem[24]~[27] and the lack of convincing and simple prescription to deal with it in the framework of BRST symmetry. In the next Section I would like to discuss this issue in some detail.

## 2 BRST invariant path integral in the presence of Gribov copies

There have been many attempts to provide a simple prescription to quantize the theory with Gribov problems. As one of these attempts to quantize the theory with Gribov

copies by using BRST symmetry[28][29], we here recapitulate the essence of the argument presented in [28]. We start with the Faddeev-Popov formulation of the Feynman-type gauge condition [30]. The vacuum-to-vacuum transition amplitude is defined by

$$\langle +\infty | -\infty \rangle = \int \mathcal{D}A_\mu^\omega \mathcal{D}C \delta(\partial^\mu A_\mu^\omega - C) \Delta(A) \exp\{iS(A_\mu^\omega) - \frac{i}{2\alpha} \int C(x)^2 dx\} \quad (2.1)$$

where  $S(A_\mu^\omega)$  stands for the action invariant under the Yang-Mills local gauge transformation. The positive constant  $\alpha$  is a gauge fixing parameter which specifies the Feynman-type gauge condition. In the following we often suppress the internal symmetry indices, and instead we write the gauge parameter explicitly:  $A_\mu^\omega$  indicates the gauge field which is obtained from  $A_\mu$  by a gauge transformation specified by  $\omega(x)$ . The determinant factor  $\Delta(A)$  is defined by[1][30]

$$\begin{aligned} \Delta(A)^{-1} &= \int \mathcal{D}\omega \mathcal{D}C \delta(\partial^\mu A_\mu^\omega - C) \exp\{-\frac{i}{2\alpha} \int C(x)^2 dx\} \\ &\approx \text{const} \left\{ \sum_k |\det[\frac{\partial}{\partial \omega_k} \partial^\mu A_\mu^{\omega_k}]|^{-1} \right\} \end{aligned} \quad (2.2)$$

where the summation runs over all the gauge equivalent configurations satisfying  $\partial^\mu A_\mu^{\omega_k} = 0$ , which were found by Gribov[24] and others[25]~[27]. Equation(2.2) is valid only for sufficiently small  $\alpha$ , since the parameter  $\omega'(\omega, A, C)$  defined by

$$\partial^\mu A_\mu^{\omega'(\omega, A, C)} = \partial^\mu A_\mu^\omega - C \quad (2.3)$$

has a complicated branch structure for large  $C$  in the presence of Gribov ambiguities. Obviously the Feynman-type gauge formulation becomes even more involved than the Landau-type gauge condition.

It was suggested in [28] to replace equation (2.1) by

$$\langle +\infty | -\infty \rangle = \frac{1}{N} \int \mathcal{D}A_\mu^\omega \mathcal{D}C \delta(\partial^\mu A_\mu^\omega - C) \det[\frac{\partial}{\partial \omega} \partial^\mu A_\mu^\omega] \exp\{iS(A_\mu^\omega) - \frac{i}{2\alpha} \int C(x)^2 dx\} \quad (2.4)$$

The crucial difference between (2.1) and (2.4) is that (2.4) is local in the gauge space  $\omega(x)$  (i.e., the gauge fixing factor and the compensating factor are defined at the identical  $\omega$ ), whereas  $\Delta(A)$  in (2.1) is gauge independent and involves a non-local factor in  $\omega$  as is shown in (2.2). As the determinant in (2.4) depends on  $A_\mu^\omega$ , the entire integrand in (2.4) is in general no more degenerate with respect to gauge equivalent configurations even if the

gauge fixing term itself may be degenerate for certain configurations. Another important point is that one takes the absolute values of determinant factors in (2.2) thanks to the definition of the  $\delta$ -function, whereas just the determinant which can be negative as well as positive appears in (2.4).

It is easy to see that (2.4) can be rewritten as

$$\langle +\infty | -\infty \rangle = \frac{1}{\tilde{N}} \int \mathcal{D}A_\mu^\omega \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c \exp\{iS(A_\mu^\omega) + i \int \mathcal{L}_g dx\} \quad (2.5)$$

where

$$\mathcal{L}_g = -\partial^\mu B^a A_\mu^{a\omega} + i\partial^\mu \bar{c}^a (\partial_\mu - g f^{abc} A_\mu^{b\omega}) c^c + \frac{\alpha}{2} B^a B^a \quad (2.6)$$

with  $B^a$  the Lagrangian multiplier field, and  $\bar{c}^a$  and  $c^a$  the (hermitian) Faddeev-Popov ghost fields;  $f^{abc}$  is the structure constant of the gauge group and  $g$  is the gauge coupling constant. If one imposes the hermiticity of  $\bar{c}^a$  and  $c^a$ , the phase factor of the determinant in (2.4) cannot be removed. The normalization constant  $\tilde{N}$  in (2.5) includes the effect of Gaussian integral over  $B$  in addition to  $N$  in (2.4), and in fact  $\tilde{N}$  is independent of  $\alpha$ . See eq. (2.10). This  $\mathcal{L}_g$  as well as the starting gauge invariant Lagrangian are invariant under the BRST transformation defined by

$$\begin{aligned} \delta_\theta A_\mu^{a\omega} &= i\theta[\partial_\mu c^a - g f^{abc} A_\mu^{b\omega} c^c] \\ \delta_\theta c^a &= i\theta(g/2) f^{abc} c^b c^c \\ \delta_\theta \bar{c}^a &= \theta B^a \\ \delta_\theta B^a &= 0 \end{aligned} \quad (2.7)$$

where  $\theta$  and the ghost variables  $c^a(x)$  and  $\bar{c}^a(x)$  are elements of the Grassmann algebra, i.e.,  $\theta^2 = 0$ . This transformation can be confirmed to be nil-potent  $\delta^2 = 0$ , for example,

$$\delta_{\theta_2}(\delta_{\theta_1} A_\mu^{a\omega}) = 0 \quad (2.8)$$

One can also confirm that the path integral measure in (2.5) is invariant under (2.7). Note that the transformation (2.7) is “local” in the  $\omega$  parameter; precisely for this property, the prescription in (2.4) was chosen.

To interpret the path integral measure in (2.4) as the path integral over all the gauge field configurations divided by the gauge volume, namely

$$\langle +\infty | -\infty \rangle = \int \frac{\mathcal{D}A_\mu^\omega}{\text{gauge volume}(\omega)} \exp\{iS(A_\mu^\omega)\} \quad (2.9)$$

one needs to define the normalization factor in (2.4) by

$$\begin{aligned}
N &= \int \mathcal{D}\omega \mathcal{D}C \delta(\partial^\mu A_\mu^\omega - C) \det\left[\frac{\partial}{\partial\omega} \partial^\mu A_\mu^\omega\right] \exp\left\{-\frac{i}{2\alpha} \int C(x)^2 dx\right\} \\
&= \int \mathcal{D}\omega \det\left[\frac{\partial}{\partial\omega} \partial^\mu A_\mu^\omega\right] \exp\left\{-\frac{i}{2\alpha} \int (\partial^\mu A_\mu^\omega)^2 dx\right\} \\
&= \int \mathcal{D}\tau \exp\left\{-\frac{i}{2\alpha} \int \tau(x)^2 dx\right\}
\end{aligned} \tag{2.10}$$

where the function  $\tau$  is defined by

$$\tau(x) \equiv \partial^\mu A_\mu^\omega(x) \tag{2.11}$$

and the determinant factor is regarded as a Jacobian for the change of variables from  $\omega(x)$  to  $\tau(x)$ . Although we use the Feynman-type gauge fixing (2.11) as a typical example in this Section, one may replace (2.11) by

$$\tau(x) \equiv f(A_\mu^\omega(x)) \tag{2.12}$$

to deal with a more general gauge condition

$$f(A_\mu^\omega(x)) = 0 \tag{2.13}$$

It is crucial to establish that the normalization factor in (2.10) is independent of  $A_\mu$ . Only in this case, (2.4) defines an acceptable vacuum transition amplitude. The Gribov ambiguity in the present case appears as a non-unique correspondence between  $\tau(x)$  and  $\omega(x)$  in (2.11), as is schematically shown in Fig. 1 which includes 3 Gribov copies. The path integral in (2.10) is performed along the contour in Fig. 1. As the Gaussian function is regular at any finite point, the complicated contour in Fig. 1 gives rise to the same result in (2.10) as a contour corresponding to  $A_\mu = 0$ . In the present path integral formulation, the evaluation of the normalization factor in (2.10) is the only place where we explicitly encounter the multiple solutions of gauge fixing condition.[ If the normalization factor  $N$  should depend on gauge field  $A_\mu$ , the factor  $N$ , which is gauge independent in the sense that we integrated over entire gauge orbit, needs to be taken inside the path integral in (2.4). In this case one loses the simplicity of the formula (2.4).]

The basic assumption we have to make is therefore that (2.11) in the context of the path integral (2.10) is “globally single-valued”, in the sense that the asymptotic functional correspondence between  $\omega$  and  $\tau$  is little affected by a fixed  $A_\mu$  with  $\partial^\mu A_\mu = 0$  [28];

Fig.1 satisfies this requirement. This assumption appears to be physically reasonable if the second derivative term of the gauge orbit parameter dominates the functional correspondence in (2.11), though it has not been established mathematically. To define the functional correspondence between  $\omega$  and  $\tau$  in (2.11), one needs in general some notion of norm such as  $L^2$ -norm for which the Coulomb gauge vacuum is unique [26][27]. The functional configurations which are square integrable however have zero measure in the path integral[31], and this makes the precise analysis of (2.11) very complicated: At least what we need to do is to start with an expansion of a generic field variable in terms of some complete orthonormal basis set (which means that the field is inside the  $L^2$ -space) and then let each expansion coefficient vary from  $-\infty$  to  $+\infty$ (which means that the field is outside the  $L^2$ -space).

The indefinite signature of the determinant factor in (2.4) is not a difficulty in the framework of indefinite metric field theory [11]~[15] since the determinant factor is associated with the Faddeev-Popov ghost fields and the BRST cohomology selects the positive definite physical space. On the other hand, the Gribov problem may also suggest that one cannot bring the relation(2.11) with fixed  $A_\mu$  to  $\partial^\mu A_\mu^\omega = 0$  by any gauge transformation [25]. If this is the case, the asymptotic behavior of the mapping (2.11) is in general modified by  $A_\mu$  and our prescription cannot be justified.

It is not known at this moment to what extent the global single-valuedness of (2.11) in the sense of the path integral (2.10) is satisfied by 4-dimensional Yang-Mills fields. It is therefore very important to deepen our understanding of this problem by studying a simpler model with a finite number of degrees of freedom. Recently, a very detailed analysis of a soluble gauge model with Gribov-type copies has been performed by Friedberg, Lee, Pang, and Ren[32]. As is explained below, their model nicely satisfies our criterion in (2.11). Although it is not clear whether the properties exhibited by this soluble model are generic for theories with Gribov copies, it is expected that the soluble model will help increase our understanding of the problem and may provide us a deeper insight into the possible quantization of gauge theories.



### 3 A soluble gauge model with Gribov-type copies

#### 3.1, THE MODEL OF FRIEDBERG, LEE, PANG, AND REN

The soluble gauge model of Friedberg, Lee, Pang and Ren[32] is defined by

$$\mathcal{L} = \frac{1}{2} \{ [\dot{X}(t) + g\xi(t)Y(t)]^2 + [\dot{Y}(t) - g\xi(t)X(t)]^2 + [\dot{Z}(t) - \xi(t)]^2 \} - U(X(t)^2 + Y(t)^2) \quad (3.1)$$

where  $\dot{X}(t)$ , for example, means the time derivative of  $X(t)$ , and the potential  $U$  depends only on the combination  $X^2 + Y^2$ . This Lagrangian is invariant under a local gauge transformation parametrized by  $\omega(t)$ ,

$$\begin{aligned} X^\omega(t) &= X(t) \cos g\omega(t) - Y(t) \sin g\omega(t) \\ Y^\omega(t) &= X(t) \sin g\omega(t) + Y(t) \cos g\omega(t) \\ Z^\omega(t) &= Z(t) + \omega(t) \\ \xi^\omega(t) &= \xi(t) + \dot{\omega}(t) \end{aligned} \quad (3.2)$$

The gauge condition ( an analogue of  $A_0 = 0$  gauge )

$$\xi(t) = 0 \quad (3.3)$$

or ( an analogue of  $A_3 = 0$  gauge )

$$Z(t) = 0 \quad (3.4)$$

is well-defined without suffering from Gribov-type copies. However, it was shown in [32] that the gauge condition ( an analogue of the Coulomb gauge )

$$Z(t) - \lambda X(t) = 0 \quad (3.5)$$

with a constant  $\lambda$  suffers from the Gribov- type complications. This is seen by using the notation in (3.2) as

$$\begin{aligned} Z^\omega(t) - \lambda X^\omega(t) &= Z(t) + \omega(t) - \lambda X(t) \cos g\omega(t) + \lambda Y(t) \sin g\omega(t) \\ &= \omega(t) + \lambda \sqrt{X^2 + Y^2} [\cos \phi(t) - \cos(g\omega(t) + \phi(t))] = 0 \end{aligned} \quad (3.6)$$

where we used the relation(3.5) and

$$X(t) = \sqrt{X^2 + Y^2} \cos \phi(t), \quad Y(t) = \sqrt{X^2 + Y^2} \sin \phi(t) \quad (3.7)$$

From a view point of gauge fixing,  $\omega(t) = 0$  is a solution of (3.6) if (3.5) is satisfied. By analyzing the crossing points of two graphs in  $(\omega, \eta)$  plane defined by

$$\begin{aligned}\eta &= \frac{1}{\lambda\sqrt{X^2 + Y^2}}\omega \\ \eta &= \cos(g\omega + \phi) - \cos\phi\end{aligned}\tag{3.8}$$

one can confirm that eq.(3.6) in general has more than one solutions for  $\omega$ .

From a view point of general gauge fixing procedure, we here regard the algebraic gauge fixing such as (3.3) and (3.4) well-defined; in the analysis of the Gribov problem in Ref.[25], the algebraic gauge fixing [26] is excluded.

The authors in Ref.[32] started with the Hamiltonian formulated in terms of the well-defined gauge  $\xi(t) = 0$  in (3.3) and then faithfully rewrote the Hamiltonian in terms of the variables defined by the ‘‘Coulomb gauge’’ in (3.5). By this way, the authors in [32] analyzed in detail the problem related to the Gribov copies and the so-called Gribov horizons where the Faddeev-Popov determinant vanishes. They thus arrived at a prescription which sums over all the Gribov-type copies in a very specific way. As is clear from their derivation, their specification satisfies the unitarity and gauge independence.

In the context of BRST invariant path integral discussed in Section 2, the crucial relation (2.11) becomes

$$\begin{aligned}\tau(t) &= Z^\omega(t) - \lambda X^\omega(t) \\ &= \omega(t) + Z(t) - \lambda X(t) \cos g\omega(t) + \lambda Y(t) \sin g\omega(t)\end{aligned}\tag{3.9}$$

in the present model. For  $X = Y = 0$ , the functional correspondence between  $\omega$  and  $\tau$  is one-to-one and monotonous for any fixed value of  $t$ . When one varies  $X(t), Y(t)$  and  $Z(t)$  continuously, one deforms this monotonous curve continuously. But the asymptotic correspondence between  $\omega(t)$  and  $\tau(t)$  at  $\omega(t) = \pm\infty$  for each value of  $t$  is still kept preserved, at least for any fixed  $X(t), Y(t)$  and  $Z(t)$ . This correspondence between  $\omega(t)$  and  $\tau(t)$  thus satisfies our criterion discussed in connection with (2.11). The absence of terms which contain the derivatives of  $\omega(t)$  in (3.9) makes the functional correspondence in (3.9) well-defined and transparent.

From a view point of gauge fixing in (3.6), this ‘‘globally single-valued’’ correspondence between  $\omega$  and  $\tau$  means that one always obtains an *odd* number of solutions for (3.6). The

prescription in [32] is then viewed as a sum of all these solutions with signature factors specified by the signature of the Faddeev-Popov determinant

$$\det\left\{\frac{\partial}{\partial\omega(t')}[Z^\omega(t) - \lambda X^\omega(t)]\right\} = \det\{[1 + \lambda g Y^\omega(t)]\delta(t - t')\} \quad (3.10)$$

evaluated at the point of solutions ,  $\omega = \omega(\sqrt{X^2 + Y^2}, \phi)$ , of (3.6). The row and column indices of the matrix in (3.10) are specified by  $t$  and  $t'$ , respectively. In the context of BRST invariant formulation, a pair-wise cancellation of Gribov-type copies takes place, except for one solution, in the calculation of the normalization factor in (2.10) or (3.21) below.

### 3.2, BRST INVARIANT PATH INTEGRAL

The relation (3.9) satisfies our criterion discussed in connection with (2.11). We can thus define an analogue of (2.5) for the Lagrangian (3.1) by

$$\langle +\infty | -\infty \rangle = \frac{1}{N} \int d\mu \exp\{iS(X^\omega, Y^\omega, Z^\omega, \xi^\omega) + i \int \mathcal{L}_g dt\} \quad (3.11)$$

where

$$\begin{aligned} S(X^\omega, Y^\omega, Z^\omega, \xi^\omega) &= \int \mathcal{L}(X^\omega, Y^\omega, Z^\omega, \xi^\omega) dt \\ &= S(X, Y, Z, \xi) \end{aligned} \quad (3.12)$$

in terms of the Lagrangian  $\mathcal{L}$  in (3.1). The gauge fixing part of (3.11) is defined by

$$\mathcal{L}_g = -\beta \dot{B} \xi^\omega + B(Z^\omega - \lambda X^\omega) + \beta i \bar{c} \dot{c} - i \bar{c}(1 + g \lambda Y^\omega) c + \frac{\alpha}{2} B^2 \quad (3.13)$$

where  $\alpha, \beta$  and  $\lambda$  are numerical constants, and  $\bar{c}$  and  $c$  are (hermitian) Faddeev-Popov ghost fields.  $B$  is a Lagrangian multiplier field. Note that  $\mathcal{L}_g$  is hermitian. The integral measure in (3.11) is given by

$$d\mu = \mathcal{D}X^\omega \mathcal{D}Y^\omega \mathcal{D}Z^\omega \mathcal{D}\xi^\omega \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c \quad (3.14)$$

The Lagrangians  $\mathcal{L}$  and  $\mathcal{L}_g$  and the path integral measure (3.14) are invariant under the BRST transformation defined by

$$\begin{aligned} X^\omega(t, \theta) &= X^\omega(t) - i\theta g c(t) Y^\omega(t) \\ Y^\omega(t, \theta) &= Y^\omega(t) + i\theta g c(t) X^\omega(t) \end{aligned}$$

$$\begin{aligned}
Z^\omega(t, \theta) &= Z^\omega(t) + i\theta c(t) \\
\xi^\omega(t, \theta) &= \xi^\omega(t) + i\theta \dot{c}(t) \\
c(t, \theta) &= c(t) \\
\bar{c}(t, \theta) &= \bar{c}(t) + \theta B(t)
\end{aligned} \tag{3.15}$$

where the parameter  $\theta$  is a Grassmann number,  $\theta^2 = 0$ . Note that  $\theta$  and ghost variables anti-commute. In (3.15) we used a BRST superfield notation: In this notation, the second component of a superfield proportional to  $\theta$  stands for the BRST transformed field of the first component. The second component is invariant under BRST transformation which ensures the nil-potency of the BRST charge. In the operator notation to be defined later, one can write , for example,

$$X^\omega(t, \theta) = e^{-\theta Q} X^\omega(t, 0) e^{\theta Q} \tag{3.16}$$

with a nil-potent BRST charge  $Q$ ,  $\{Q, Q\}_+ = 0$ . Namely, the BRST transformation is a translation in  $\theta$ -space, and  $\theta Q$  is analogous to momentum operator.

In (3.11)~(3.15), we explicitly wrote the gauge parameter  $\omega$  to emphasize that BRST transformation is “local” in the  $\omega$ -space. For  $\mathcal{L}_g$  in (3.13), the relation (3.9) is replaced by ( an analogue of the Landau gauge )

$$\begin{aligned}
\tau(t) &\equiv \beta \dot{\xi}^\omega(t) + Z^\omega(t) - \lambda X^\omega(t) \\
&= \beta \ddot{\omega}(t) + \omega(t) + \beta \dot{\xi}(t) + Z(t) - \lambda X(t) \cos g\omega(t) + \lambda Y(t) \sin g\omega(t)
\end{aligned} \tag{3.17}$$

The functional correspondence between  $\omega$  and  $\tau$  is monotonous and one-to-one for weak  $\xi(t), Z(t), X(t)$  and  $Y(t)$  fields; this is understood if one rewrites the relation (3.17) for Euclidean time  $t = -it_E$  by neglecting weak fields as

$$\tau(t_E) = (-\beta \frac{d^2}{dt_E^2} + 1) \omega(t_E)$$

The Fourier transform of this relation gives a one-to-one monotonous correspondence between the Fourier coefficients of  $\tau$  and  $\omega$  for non-negative  $\beta$ . The asymptotic functional correspondence between  $\omega$  and  $\tau$  for weak field cases is preserved even for any fixed strong fields  $\xi(t), Z(t), X(t)$  and  $Y(t)$  for non-negative  $\beta$  to the extent that the term linear in  $\omega(t)$  dominates the cosine and sine terms. The correspondence between  $\tau$  and  $\omega$  in (3.17) is quite complicated for finite  $\omega(t)$  due to the presence of the derivatives of  $\omega(t)$ .

Thus (3.17) satisfies our criterion of BRST invariant path integral for any non-negative  $\beta$ . The relation (3.9) is recovered if one sets  $\beta = 0$  in (3.17); the non-zero parameter  $\beta \neq 0$  however renders a canonical structure of the theory better-defined. For example, the kinetic term for ghost fields in (3.13) disappears for  $\beta = 0$ . In this respect the gauge (3.5) is also analogous to the unitary gauge. In the following we set  $\beta = \alpha > 0$  in (3.13),

$$\mathcal{L}_g = -\alpha \dot{B} \xi^\omega + B(Z^\omega - \lambda X^\omega) + \alpha i \dot{\bar{c}} \dot{c} - i \bar{c}(1 + g\lambda Y^\omega)c + \frac{\alpha}{2} B^2 \quad (3.18)$$

and let  $\alpha \rightarrow 0$  later. In the limit  $\alpha = 0$ , one recovers the gauge condition (3.5) defined in Ref.[32]. This procedure is analogous to  $R_\xi$ -gauge ( or the  $\xi$ -limiting process of Lee and Yang [33] ), where the (singular) unitary gauge is defined in the vanishing limit of the gauge parameter,  $\xi \rightarrow 0$ : In (3.18) the parameter  $\alpha$  plays the role of  $\xi$  in  $R_\xi$ - gauge.

By using the BRST invariance, one can show the  $\lambda$ - independence of (3.11) as follows:  
 $\langle +\infty | -\infty \rangle_{\lambda+\delta\lambda} = \langle +\infty | -\infty \rangle_\lambda - \delta\lambda \frac{1}{N} \int d\mu [B(t)X^\omega(t) + ig\bar{c}(t)Y^\omega(t)c(t)] \exp\{i \int (\mathcal{L} + \mathcal{L}_g) dt\}$  (3.19)

where we perturbatively expanded in the variation of  $\mathcal{L}_g$  for a change of the parameter  $\lambda + \delta\lambda$ ,

$$\mathcal{L}_g(\lambda + \delta\lambda) = \mathcal{L}_g(\lambda) - \delta\lambda [B(t)X^\omega(t) + ig\bar{c}(t)Y^\omega(t)c(t)] \quad (3.20)$$

This expansion is justified since the normalization factor defined by (see eq.(2.10))

$$N = \int \mathcal{D}\tau \exp\left\{-\frac{i}{2\alpha} \int \tau(x)^2 dt\right\} \quad (3.21)$$

is independent of  $\lambda$  provided that the global single-valuedness in (3.17) is satisfied. As was noted before, this path integral for  $N$ , which depends on  $\alpha$ , is the only place where we explicitly encounter the Gribov-type copies in the present approach. By denoting the BRST transformed variables by prime, for example,

$$X^\omega(t)' = X^\omega(t) - ig\theta c(t)Y^\omega(t) \quad (3.22)$$

we have a BRST identity ( or Slavnov-Taylor identity [2])

$$\begin{aligned} & \frac{1}{N} \int d\mu \bar{c}(t) X^\omega(t) \exp\{i \int (\mathcal{L} + \mathcal{L}_g) dt\} \\ &= \frac{1}{N} \int d\mu' \bar{c}(t)' X^\omega(t)' \exp\{i \int (\mathcal{L}' + \mathcal{L}_g') dt\} \\ &= \frac{1}{N} \int d\mu \bar{c}(t) X^\omega(t) \exp\{i \int (\mathcal{L} + \mathcal{L}_g) dt\} \\ &+ \theta \frac{1}{N} \int d\mu [B(t)X^\omega(t) + ig\bar{c}(t)Y^\omega(t)c(t)] \exp\{i \int (\mathcal{L} + \mathcal{L}_g) dt\} \end{aligned} \quad (3.23)$$

where the first equality holds since the path integral is independent of the naming of integration variables provided that the asymptotic behavior and the boundary conditions are not modified by the change of variables. The second equality in (3.23) holds due to the BRST invariance of the measure and the action

$$\begin{aligned} d\mu' &= d\mu, \\ \mathcal{L}' + \mathcal{L}'_g &= \mathcal{L} + \mathcal{L}_g \end{aligned} \quad (3.24)$$

but

$$\bar{c}(t)' X^\omega(t)' = \bar{c}(t) X^\omega(t) + \theta[B(t) X^\omega(t) + ig\bar{c}(t) Y^\omega(t) c(t)] \quad (3.25)$$

From (3.23) one concludes

$$\frac{1}{N} \int d\mu [B(t) X^\omega(t) + ig\bar{c}(t) Y^\omega(t) c(t)] \exp\{i \int (\mathcal{L} + \mathcal{L}_g) dt\} = 0 \quad (3.26)$$

and thus

$$\langle +\infty | -\infty \rangle_{\lambda+\delta\lambda} = \langle +\infty | -\infty \rangle_\lambda \quad (3.27)$$

in (3.19). This relation shows that the ground state energy is independent of the parameter  $\lambda$ ; in particular one can choose  $\lambda = 0$  in evaluating the ground state energy, which leads to the gauge condition (3.4) without Gribov complications.

In the path integral (3.11), one may impose *periodic* boundary conditions in time  $t$  on all the integration variables and let the time interval  $\rightarrow \infty$  later so that BRST transformation (3.15) be consistent with the boundary conditions.

The analysis in this sub-section is general but formal. In the next sub-section we comment on an operator Hamiltonian formalism and BRST cohomology.

### 3.3, BRST COHOMOLOGY

We start with the BRST invariant effective Lagrangian

$$\begin{aligned} \mathcal{L}_{eff} &= \mathcal{L} + \mathcal{L}_g \\ &= \frac{1}{2} \{ [\dot{X}^\omega(t) + g\xi(t)^\omega Y^\omega(t)]^2 + [\dot{Y}^\omega(t) - g\xi^\omega(t) X^\omega(t)]^2 + [\dot{Z}^\omega(t) - \xi^\omega(t)]^2 \} \\ &\quad - U[(X^\omega(t))^2 + (Y^\omega(t))^2] \\ &\quad - \alpha \dot{B}(t) \xi^\omega(t) + B(t) (Z^\omega(t) - \lambda X^\omega(t)) + \alpha i \dot{\bar{c}}(t) \dot{c}(t) \\ &\quad - i \bar{c}(t) [1 + g\lambda Y^\omega(t)] c(t) + \frac{\alpha}{2} B(t)^2 \end{aligned} \quad (3.28)$$

obtained from (3.1) and (3.18). A justification of (3.28), in particular its treatment of Gribov-type copies, rests on the path integral representation (3.11). In the following we suppress the suffix  $\omega$ , which emphasizes that the BRST transformation is local in  $\omega$ -space.

One can construct a Hamiltonian from (3.28) in a standard manner as

$$H = \frac{1}{2}[P_X^2 + P_Y^2 + P_Z^2] + U(X^2 + Y^2) + \xi G - B(Z - \lambda X) + i\frac{1}{\alpha}p_c p_{\bar{c}} + i\bar{c}(1 + \lambda g Y)c - \frac{1}{2}\alpha B^2 \quad (3.29)$$

and the Gauss operator is given by

$$G \equiv g(XP_Y - YP_X) + P_Z \quad (3.30)$$

We note that  $(p_{\bar{c}})^\dagger = -p_{\bar{c}}$  since  $p_{\bar{c}} = i\alpha\dot{c}$  and  $p_c = -i\alpha\dot{\bar{c}}$ .

The BRST charge is obtained from  $\mathcal{L}_{eff}$  (3.28) via the Noether current as

$$Q = cG - ip_{\bar{c}}B \quad (3.31)$$

The BRST charge  $Q$  is hermitian  $Q^\dagger = Q$  and nil-potent

$$\{Q, Q\}_+ = 0 \quad (3.32)$$

by noting  $\{c, c\}_+ = \{p_{\bar{c}}, p_{\bar{c}}\}_+ = \{p_{\bar{c}}, c\}_+ = 0$ . The BRST transformation (3.15) is generated by  $Q$ , for example,

$$\begin{aligned} e^{-\theta Q}X(t)e^{\theta Q} &= X(t) - [\theta Q, X(t)] \\ &= X(t) - i\theta g c(t)Y(t), \\ e^{-\theta Q}\bar{c}(t)e^{\theta Q} &= \bar{c}(t) - [\theta Q, \bar{c}(t)] \\ &= \bar{c}(t) + \theta B(t) \end{aligned} \quad (3.33)$$

by noting  $\theta^2 = 0$ .

The Hamiltonian in (3.29) is rewritten by using the BRST charge as

$$H = H_0 + i\{Q, \xi p_c\}_+ + \{Q, \bar{c}(Z - \lambda X)\}_+ + \frac{\alpha}{2}\{Q, B\bar{c}\}_+ \quad (3.34)$$

with

$$H_0 \equiv \frac{1}{2}[P_X^2 + P_Y^2 + P_Z^2] + U(X^2 + Y^2) \quad (3.35)$$

The physical state  $\Psi$  is defined as an element of BRST cohomology

$$\Psi \in \text{Ker } Q / \text{Im } Q \quad (3.36)$$

namely

$$Q\Psi = 0 \quad (3.37)$$

but  $\Psi$  is *not* written in a form  $\Psi = Q\Phi$  with a non-vanishing  $\Phi$ .

The time development of  $\Psi$  is dictated by Schroedinger equation

$$i\frac{\partial}{\partial t}\Psi(t) = H\Psi(t) \quad (3.38)$$

and thus

$$\begin{aligned} \Psi(\delta t) &= e^{-iH\delta t}\Psi(0) \\ &= \Psi(0) - i\delta t H\Psi(0) \\ &= \Psi(0) - i\delta t H_0\Psi(0) \\ &\quad - i\delta t Q\{i\xi p_c + \bar{c}(Z - \lambda X) + \frac{\alpha}{2}B\bar{c}\}\Psi(0) \\ &\simeq \Psi(0) - i\delta t H_0\Psi(0) \end{aligned} \quad (3.39)$$

in the sense of BRST cohomology by noting (3.34) and  $Q\Psi(0) = 0$ . Note that the Hamiltonian is BRST invariant

$$[Q, H_0] = [Q, H] = 0 \quad (3.40)$$

If one solves the time independent Schroedinger equation

$$H_0\Psi(0) = E\Psi(0) \quad (3.41)$$

with  $Q\Psi(0) = 0$ , one obtains

$$\begin{aligned} \Psi(0)^\dagger e^{-iHt}\Psi(0) &= e^{-iEt}\Psi(0)^\dagger\Psi(0) \\ &= e^{-iEt} \end{aligned} \quad (3.42)$$

by noting  $\Psi(0)^\dagger Q = 0$ . The eigen-value equation (3.41) is gauge independent and thus  $E$  is formally gauge independent.



In fact, a more detailed analysis[34] confirms that the BRST cohomology reproduces the result of canonical analysis in Ref.[32]. Namely, one can show that the physical spectrum of the soluble model with the harmonic potential

$$U(X^2 + Y^2) = \frac{\omega^2}{2}(X^2 + Y^2) \quad (3.43)$$

is given by

$$H = \omega(n_1 + n_2 + 1) + \frac{g^2}{2}(n_1 - n_2)^2 \quad (3.44)$$

where  $n_1$  and  $n_2$  are non-negative integers. The first term in this formula stands for the spectrum of a two-dimensional harmonic oscillator, and the second term stands for  $\frac{1}{2}P_Z^2$  replaced by  $\frac{g^2}{2}L_Z^2$  by using the Gauss law operator in (3.30). One can safely take the limit  $\alpha \rightarrow 0$  in the physical sector, though unphysical excitations acquire infinite excitation energy in this limit just like unphysical excitations in gauge theory defined by  $R_\xi$ -gauge[33].

It has been shown in [32] that the correction terms arising from operator ordering plays a crucial role in the evaluation of perturbative corrections to ground state energy in Lagrangian path integral formula. This problem is often treated casually in conventional perturbative calculations; a general belief (and hope) is that Lorentz invariance and BRST invariance somehow takes care of the operator ordering problem. It is shown[34] that BRST invariance and  $T^*$ -product prescription reproduce the correct result of Ref.[32] provided that one uses a canonically well-defined gauge such as  $R_\xi$ -gauge with  $\alpha \neq 0$  in (3.18). This check is important to establish the equivalence of (3.11) to the path integral formula in Ref.[32]. If one starts with  $\alpha = 0$  from the on-set, one needs correction terms calculated in Ref.[32].

To be more explicit, what we want to evaluate is eq.(3.11), namely

$$\langle +\infty | -\infty \rangle = \frac{1}{N} \int d\mu \exp\{iS(X^\omega, Y^\omega, Z^\omega, \xi^\omega) + i \int \mathcal{L}_g dt\} \quad (3.45)$$

We define the path integral for a sufficiently large time interval

$$t \in [T/2, -T/2] \quad (3.46)$$

and let  $T \rightarrow \infty$  later.

The exact ground state energy is given by eq.(3.44) as

$$E = \omega \tag{3.47}$$

Namely, we have no correction depending on the gauge parameter  $\lambda$  and the coupling constant  $g$ . As was already shown in (3.27), the absence of  $\lambda$  dependence is a result of BRST symmetry. This property is thus more general and, in fact, it holds for all the energy spectrum of physical states; this can be shown by using the Schwinger's action principle [35] and the definition of physical states in (3.36). The perturbative check of  $\lambda$ -independence or Slavnov-Taylor identities in general is carried out in the standard manner. On the other hand, the absence of  $g$ -dependence is an effect of more dynamical origin and it can be confirmed by an explicit calculation[34]: The ground state energy is then obtained from

$$\begin{aligned} \langle +\infty | -\infty \rangle &= \lim_{T \rightarrow \infty} \langle 0 | e^{-iH[T/2 - (-T/2)]} | 0 \rangle \\ &= \lim_{T \rightarrow \infty} \text{const} \times e^{-i\omega T} \end{aligned} \tag{3.48}$$

which is justified for  $T = -iT_E$  and  $T_E \rightarrow \infty$  in Euclidean theory. We thus obtain the ground state energy  $E = \omega$  to be consistent with (3.47).

## 4 Discussion and conclusion

The BRST symmetry plays a central role in modern gauge theory, and the BRST invariant path integral can be formulated by summing over all the Gribov-type copies in a very specific manner provided that the crucial correspondence in (2.11) or (3.17) is globally single valued[28]. This criterion is satisfied by the soluble gauge model proposed in Ref.[32], and it is encouraging that the BRST invariant prescription is in accord with the canonical analysis of the soluble gauge model in Ref.[32]. The detailed explicit analysis in Ref.[32] and the somewhat formal BRST analysis are complementary to each other. In Ref.[32], the problem related to the so-called Gribov horizon, in particular the possible singularity associated with it, has been analyzed in greater detail; this is crucial for the analysis of more general situation. On the other hand, an advantage of the BRST analysis is that one can clearly see the gauge independence of physical quantities such as the energy spectrum as a result of BRST identity.

The BRST approach allows a transparent treatment of general class of gauge conditions implemented by (3.18). This gauge condition with  $\alpha \neq 0$  renders the canonical structure better-defined, and it allows simpler perturbative treatments of the problems such as the corrections to the ground state energy. The BRST analysis in Ref.[34] vis-a-vis the explicit canonical analysis in Ref.[32] may provide a justification of conventional covariant perturbation theory in gauge theory, which is based on Lorentz invariance( or  $T^*$ -product ) and BRST invariance without the operator ordering terms.

Motivated by the observation in Ref.[32] to the effect that the Gribov horizons are not really singular in quantum mechanical sense, which is in accord with our path integral in (3.21), we would like to make a speculative comment on the role of Gribov copies in QCD. Some of the non-perturbative effects such as quark confinement and hadron spectrum may be analyzed at least qualitatively in the  $1/N$  expansion scheme, for example, without referring to non-trivial topological structures of gauge fields[36]. This scheme is based on a sum of an infinite number of Feynman diagrams. This diagrammatic approach or an analytical treatment equivalent to it in the Feynman-type gauge deals with topologically trivial gauge fields but it may still suffer from the Gribov copies, as is suggested by the analysis in Ref.[27]: If one assumes that the vacuum is unique in this case as is the case in  $L^2$ -space, the global single-valuedness in (2.11) in the context of path integral (2.10) will be preserved for infinitesimally small fields  $A_\mu$ . By a continuity argument, a smooth deformation of  $A_\mu$  in  $L^2$ -space ( or its extension as explained in Section 2) will presumably keep the integral (2.10) unchanged. ( A kind of topological invariant). If this argument is valid, the formal path integral formula (2.5) will provide a basis for the analysis of some non-perturbative aspects of QCD.

On the other hand, the Gribov problem may also suggest the presence of some field configurations which do not satisfy any given gauge condition in four dimensional non-Abelian gauge theory [25]. For example, one may not be able to find any gauge parameter  $\omega(x)$  which satisfies

$$\partial^\mu A_\mu^\omega(x) = 0 \quad (4.1)$$

for some fields  $A_\mu$ . Although the measure of such field configurations in path integral is not known, the presence of such field configurations would certainly modify the asymptotic correspondence in (2.11). In the context of BRST symmetry, the Gribov problem

may then induce complicated phenomena such as the dynamical instability of BRST symmetry[37]. If the dynamical instability of BRST symmetry should take place, the relation corresponding to (3.27), which is a result of the BRST invariance of the vacuum, would no longer be derived. In the framework of path integral, this failure of (3.27) would be recognized as the failure of the expansion (3.19) since the normalization factor  $N$  in (3.21) would generally depend on not only field variables but also  $\lambda$  if the global single-valuedness in (3.17) should be violated.

In Ref.(37), a *necessary* condition for the spontaneous breakdown of BRST symmetry was formulated as

$$Tr(e^{i\pi Q_c}) = 0 \quad (4.2)$$

with  $Q_c$  the “ghost number “ operator. This relation is analogous to the Witten index in conventional supersymmetric theories[38]. It would be interesting to analyze the  $1/N$  expansion of QCD from a view point of the above index condition (4.2); in other words, if one can prove that the index is non-zero  $Tr(e^{i\pi Q_c}) \neq 0$  for the field configurations relevant for the  $1/N$  expansion, our speculation on the possible BRST symmetric treatment of some non-perturbative aspects of QCD will be justified.

Finally, we note that the lattice gauge theory [39], which is based on compactified field variables, is expected to change the scope and character of the Gribov problem completely.

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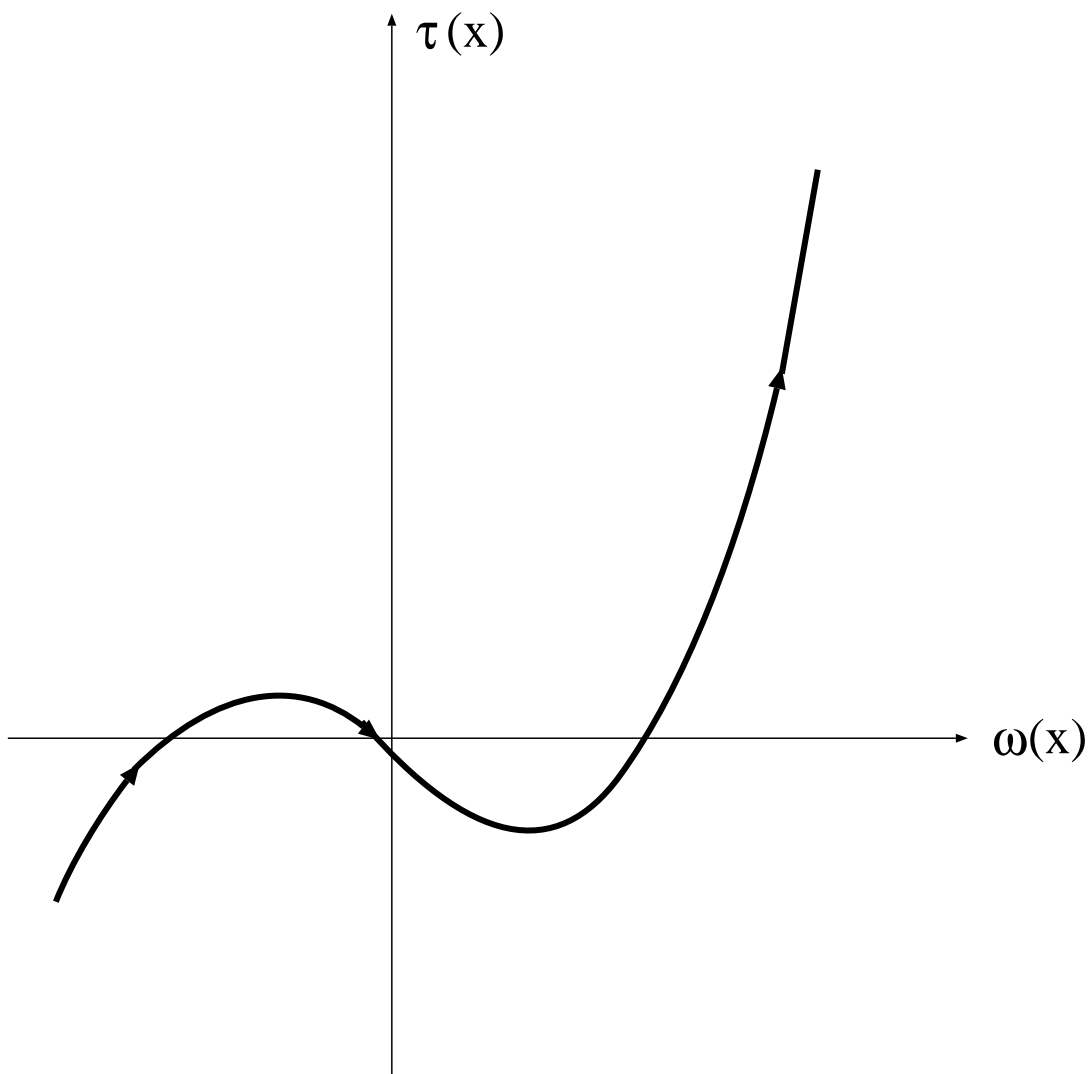


Fig. 1

Figure 1: A schematic representation of eq. (2.11) for fixed  $A_\mu$ .